AMSC/CMSC460 Computational Methods Spring 2015

Homework 5, Due on Tuesday, March 24, 2015

1. *(Mixed-type polynomial interpolation)* Let f be a function with the following point values:

x_i	1	3	4
$f(x_i)$	-3	2	4
$f'(x_i)$	1		2
$f''(x_i)$			-1

a). Find a polynomial P_5 of degree 5 that interpolates f as well as its derivatives at corresponding nodes in the table. (You do not have to simplify your result)

b). Prove that there exists $\xi = \xi(x)$ in $[1, 4]$ such that.

$$f(x) - P_5(x) = \frac{f^{(6)}(\xi)}{6!}(x-1)^2(x-3)(x-4)^3.$$

Hint: use the same idea in Theorem 6.2 and Theorem 6.4 in Suli's book.

2. *(Tridiagonal matrix algorithm)* The goal is to solve a linear system $Ax = b$, where A is a tridiagonal matrix. While traditional Gaussian elimination method requires $\mathcal{O}(n^3)$ operations, and $\mathcal{O}(n^2)$ space storage, the *Thomas algorithm* only requires $\mathcal{O}(n)$ operations, and the matrix can be stored sparsely which requires $\mathcal{O}(n)$ space storage.

a). Read the Wikipedia page for Thomas algorithm and understand how it works.
`http://en.wikipedia.org/wiki/Tridiagonal_matrix_algorithm`

b). Write a Matlab function `x = thomas(A,b)` to implement Thomas algorithm. Note that your input matrix could be sparsely defined. (You might get some help from section 2.10 in Moler's book.)

c). Download the Matlab code `testthomas.m` to the same folder where your function `thomas.m` is located. Run the script to test whether your code works or not.