AMSC/CMSC460 Computational Methods Spring 2015

Homework 1, Due on Tuesday, February 10 , 2015

1. *(Gauss elimination)* Let A be a 3-by-3 matrix given as

$$A = \begin{pmatrix} 3 & 1 & 6 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}.$$

a). Find a lower triangular matrix L_1, an upper triangular matrix U_1, such that $A = L_1 U_1$.

b). Find a lower triangular matrix L_2, an upper triangular matrix U_2 and a permutation matrix P such that $PA = L_2 U_2$.

c). Use matlab code $[\texttt{L2}, \texttt{U2}, \texttt{P}] = \texttt{lu}(\texttt{A})$ to check your answer.

d). Run matlab code $[\texttt{L3}, \texttt{U3}] = \texttt{lu}(\texttt{A})$. What is L_3? Derive L_3 by hand.

e). Solve the linear system $Ax = b$ by hand, with $b = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$.

2. *(Matlab implementation of Gauss elimination)*

a). Read through the attached Matlab code on function `mylu`. Test with A, b given in problem 1. Find an LU decomposition of A, and the solution of $Ax = b$ from the code. Check your answers with 1 a) and e).

b). Modify the code by adding pivoting to the Gauss elimination procedure. Write a Matlab function $[\texttt{x}, \texttt{LU}, \texttt{p}] = \texttt{mylup}(\texttt{A}, \texttt{b})$, where the input A is an n-by-n matrix, b is an n-vector, and the output x is the solution of $Ax = b$, LU is the matrix containing the information of LU decomposition, and p is the permutation vector. Do NOT use Matlab functions like backslash, `lu`, etc.

c). Test your code in b) on A, b given in problem 1. Check your solution with Matlab integrated functions:

- For x, check with $A \backslash b$.
- For LU, check with $\texttt{lu}(A)$.
- For p, check with the p in the output of $[L, U, p] = \texttt{lu}(A, \texttt{'vector'})$.