AMSC/CMSC460 Computational Methods Fall 2014

Homework 9-10, Due on Tuesday, December 9, 2014

1. *(Multi-step method)* Consider the following multi-step method

$$y_{n+3} = y_{n+2} + h\left(\frac{23}{12}f(x_{n+2}, y_{n+2}) - \frac{4}{3}f(x_{n+1}, y_{n+1}) + \frac{5}{12}f(x_n, y_n)\right).$$

It is known as one method in *Adams-Bashforth* family.

a). Is this method explicit or implicit?

b). Read and run the Matlab script `multistep.m`. It is a test on the method with an initial value problem:

$$y' = -y, \quad y(0) = 1.$$

The error at $x = 1$ is given numerically. What is the numerical order of accuracy of the method?

Remark: note the true solution of the system is $y(x) = e^{-x}$. In the code, $y_2 = y(h)$ and $y_3 = y(2h)$ are generated with the exact value of the solution.

c). Modify the code and generate y_2 and y_3 using forward Euler method. What numerical order of accuracy do you observe?

d). Which method can we use to get y_2 and y_3 so that there is no loss of accuracy? Propose one and test it with the code.

2. *(Boundary value problem)* Finish exercise 7.22 in Moler's book on boundary value problems of second order ODE. Read through all methods, and test on Matlab with at least 2 approaches (of your choice) to solve the ODE.

3. *(Linear regression)* Linear regression is widely used in statistics. The idea is to find a line which best fits the data. As an example, we consider the following data set.

X	1	2	4	5
Y	3	5	8	10

The goal is to find a line $y = \alpha + \beta x$ which best fits the data. More precisely, we woule like to have the least-square error:

$$\min_{\alpha,\beta} \sum_{i=1}^{4} |Y_i - (\alpha + \beta X_i)|^2.$$

a). Write the minimization problem in the standard form $\min_{\mathbf{x}} \|A\mathbf{x} - b\|_2^2$. What is $A, b, \mathbf{x}$ in this case?

b). Solve the minimization problem and generate the best line.

4. *(QR factorization)* Finish exercise 2.15 in Suli's book.