AMSC/CMSC460 Computational Methods **Fall 2014**

Homework 8, Due on Thursday, November 20, 2014

1. *(Forward Euler method)*

a). Finish exercise 12.5 in Suli's book: write down Euler method for the solution of the problem

$$y' = xe^{-5x} - 5y, \quad y(0) = 0$$

on the interval [0,1] with step size $h = 1/N$. Denoting by y_N the resulting approximation to $y(1)$. Show that $y_N \to y(1)$ as $N \to \infty$.

b). Write a Matlab code on forward Euler method. Solve the ODE in exercise 12.5 for $N = 2^s$ with $s = 1, 2, \cdots, 8$. Check the convergence rate at $x = 1$. What do you observe?

Hint: To find the error $e_s = y_N - y(1)$, you have to know the exact value of $y(1)$. For this particular problem, you can solve the ODE by hand, as it is a first order linear differential equation. To get the numerical convergence rate, take $\log_2(|e_s|)$ and find the difference between successive terms.

2. *(High order explicit method)* A trapezoid rule provides a second order one-step method for the ODE $y' = f(x, y)$. The scheme reads,

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_{n+1})].$$

However, the method is implicit. To make the method explicit, we approximate the y_{n+1} at the right hand side by forward Euler method. The new explicit scheme reads,

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f\big(x_{n+1}, y_n + hf(x_n, y_n)\big)\right].$$

a). Prove that the truncation error of this scheme is of order $\mathcal{O}(h^2)$, namely the scheme has second order accuracy.

b). Repeat problem 1 b) with the new scheme. Check if the numerical convergence rate matchs second order.