AMSC/CMSC460 Computational Methods Fall 2014

Homework 1, Due on Tuesday, September 9, 2014

1. *(Avoiding catastrophic cancellation)*
Read Chapter 1 of Moler's book, and finish exercise 1.38.

2. *(Gauss elimination and LU decomposition)*

a). Find an LU decomposition of the following matrix by hand.

$$A = \begin{pmatrix} 3 & 1 & 1 \\ -1.5 & 1.5 & -1.5 \\ 1.5 & .9 & 1.3 \end{pmatrix}.$$

b). Use matlab function `lu` to check your answer.

c). Solve the linear system $Ax = b$ using the result in a). Take $b = \begin{pmatrix} 6 \\ -3 \\ 5 \end{pmatrix}$.

3. *(Hilbert matrix)* Hilbert matrix H is defined entry-wise by $h_{ij} = \dfrac{1}{i+j-1}$.

a). Write down a 5-by-5 Hilbert matrix H.

b). Compute $\|H\|_1$ and $\|H\|_\infty$ by hand.

c). Use matlab to calculate condition number of H: κ_1, κ_2 and κ_∞. Try `cond`.

d). Compute condition numbers for 100-by-100 Hilbert matrix. Try `hilb` to create the matrix. Is the matrix well-conditioned or ill-conditioned?