CAAM 423/523 Partial Differential Equations I
MATH 423/513 Fall 2017

Homework 6, Due on Tuesday, October 31, 2017

(The problems marked with (**) are optional for everybody, and do not have to be sumitted. Solving them would require additional knowledge which is not covered in class.)

1. *(Alternative way of deriving d'Alembert's formula)* Finish problem 19 in section 2.5 of Evans book.

a). Show the general solution of the PDE $u_{xy} = 0$ is

$$u(x, y) = F(x) + G(y)$$

for arbitrary functions F, G.

b). Using the change of variables $\xi = x + ct$, $\eta = x - ct$, show $u_{tt} - c^2 u_{xx} = 0$ if and only if $u_{\xi\eta} = 0$.

c). Use (a) and (b) to rederive d'Alembert's formula.

d). Under what conditions on the initial data g, h is the solution u a right-moving wave? A left-moving wave?

2. *(Parallelogram identity)* Suppose u is a classical solution of $u_{tt} - c^2 u_{xx} = 0$. Then, for any four points that have the forms $A(x, t), B(x + ac, t - a), C(x + (a - b)c, t - a - b), D(x - bc, t - b)$, such that the parallelogram ABCD lies inside the domain, we have the following identity

$$u(A) + u(C) = u(B) + u(D).$$

Hint: It would be easy to check the identity under (ξ, η) variables, as the parallelogram becomes a rectangle.

3. (**)*(Solution for 2D wave equation)* Use the Hadamard's method of descent to find a solution for initial value problem of 2D wave equation, starting with Kirchhoff's formula for 3D wave equation. (You can find the proof in Page 73-74 of Evans book.)