CAAM/MATH423 Partial Differential Equations I **Fall 2015**

Homework 2, Due on Tuesday, September 8, 2015

1. *(Linear first order PDE)* Solve the initial value problem of the following PDE (problem 5(b) in section 3.5 of Evans book)

$$u_t + xu_x + 2yu_y = 3u, \quad u(x, y, t = 0) = g(x, y).$$

2. *(Inviscid Burgers equation)* Consider the following initial value problem

$$u_t + uu_x = 0, \quad u(x, t = 0) = \sin x.$$

a). Write down the dynamics of the characteristic paths $x(\alpha, t)$, such that

$$\frac{d}{dt}u(x(\alpha, t), t) = 0,$$

for all initial points α. Draw a sketch of the characteristic paths.

b). Prove that $x(\alpha, t) = \alpha + t \sin \alpha$. And find the condition on t such that $\alpha \to x(\alpha, t)$ is invertible.

c). Determine the time T when the characteristic structure first break down. Can you also determine the position x where the break down happens?

3. *(Initial-boundary value problem)* Find the explicit solution to the following equation on $\mathbb{R}_+ \times \mathbb{R}_+$

$$u_t + u_x = 0, \quad t \in \mathbb{R}_+, \quad x \in \mathbb{R}_+,$$

subject to initial condition

$$u(x, t = 0) = \cos x,$$

and boundary condition

$$u(x = 0, t) = 1.$$

4. *(Fully nonlinear first order PDE)* Find a smooth solution to

$$u_t + (u_x)^4 = 0, x \in \mathbb{R}, t > 0,$$

with initial condition

$$u(x, t = 0) = \frac{3}{4}x^{4/3}.$$