MATH302 Elements of Analysis

Spring 2018

Homework 7, Due on Tuesday, April 3, 2018

For all the proofs, write *in details* using standard mathematical language.

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function, and S is a closed subset of $\mathbb{R}$. Prove that $f^{-1}(S)$ is a close set.

2. Finish problem 5 in excercise 8.6 of the textbook. Show that the function f is continuous at the point a if and only if,

$$\forall\ \epsilon > 0\ \exists\ \delta > 0\ \forall\ x, y \in (a - \delta, a + \delta)\ \big(\ |f(x) - f(y)| < \epsilon\ \big).$$

3. Use the ϵ-N definition to show that the sequence (x_n) with $x_n = \frac{4n^2}{n^2+1}$ converges to 4.