MATH302 Elements of Analysis **Spring 2018**

Homework 1, Due on Thursday, January 25, 2018

For all the proofs, write *in details* using standard mathematical language.

1. Show that $(A \Leftrightarrow B) \;\Leftrightarrow\; (\neg A \Leftrightarrow \neg B)$.
Hint: use the definition of "$\Leftrightarrow$" (logically equivalent) and the truth table.

2. Prove the following statement:

$$\forall\, x > 0\ \exists\, y > 0\ (y > x \text{ and } y < 2x).$$

3. Prove that if a function f satisfies the Hölder condition with power $1/2$:

$$\exists\, C > 0\ \forall\, x \in \mathbb{R}\ \forall\, y \in \mathbb{R}\ \big(|f(x) - f(y)| \le C|x-y|^{1/2}\big),$$

then f is *pointwise continuous*, namely

$$\forall\, x \in \mathbb{R}\ \forall\, \epsilon > 0\ \exists\, \delta > 0\ \forall\, y\big(|x-y| < \delta \Rightarrow |f(x) - f(y)| < \epsilon\big).$$

Remark: one can prove a stronger statement: f is uniformly continuous, namely

$$\forall\, \epsilon > 0\ \exists\, \delta > 0\ \forall\, x, y \in \mathbb{R}\ \big(|x-y| < \delta \Rightarrow |f(x) - f(y)| < \epsilon\big).$$

See problem 1 in homework 2 for the difference between pointwise continuous and uniformly continuous.

4. Let $S = \{x \;:\; x = 5n, n = 1, 2, \cdots\}$ and $T = \{x \;:\; x = 10n + 5, n = 1, 2, \cdots\}$.

a). Show that $T \subseteq S$.

b). Show that S and T have the same cardinality.

5. Show that if A and B are in one-to-one correspondence, B and C are also in one-to-one correspondence. Then, A and C are in one-to-one correspondence.

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