# MATH141(0332/0342) Calculus II Fall 2009

**Worksheet 9, Section 9.3-9.4** Name: _____

**1.** (*3 points*) Find the Taylor polynomial of the following $f$ for the given value $n = 2$, around the point $x = 0$.

$$f(x) = \begin{cases} \frac{sin(x)}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

Hint: $f'(0) = \lim_{x \to 0} f'(x)$, if the limit exists. To calculate this, you might use L'Hopital's rule.

**2.** (*5 points*) Let $\{a_n\}_{n=1}^{\infty}$ be the sequence $\sqrt{6}, \sqrt{6+\sqrt{6}}, \sqrt{6+\sqrt{6+\sqrt{6}}}, ...$, where in general, $a_{n+1} = \sqrt{6+a_n}$.
(1) Show that for every positie integer $n$, $0 < a_n < 3$. (Notice:$\sqrt{6} < 3$)

(2) Show that for every positie integer $n$, $a_n \leq a_{n+1}$.
Hint: $a_{n+1} - a_n = a_{n+1} - (a_{n+1}^2 - 6)$. You might find the result of (1) is useful to prove (2).

(3) Show that $\{a_n\}$ converges, and find the value of $\lim_{n \to \infty} a_n$.

**3.** (*4 points*) Determine whether the following infinite sum converges or diverges. If it converges, find its sum.

(1) $\sum_{n=2}^{\infty} \frac{1}{n^2 + 2n - 3}$

(2) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$ [Hint:$\frac{1}{n(n+1)(n+2)} = \frac{1}{2}(\frac{1}{n} - \frac{1}{n+1}) - \frac{1}{2}(\frac{1}{n+1} - \frac{1}{n+2})$]

**4.** (*4 points*) Determine whether the following infinite sum converges or diverges. If it converges, find its sum.

(1) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^{2n+1}}{5^{n-2}}$

(2) $\sum_{n=1}^{\infty} \frac{3n^3 - n}{\sqrt{n+1}}$