

MATH141(0332) Calculus II

Quiz 5 - 6, October 16 - 21, 2008

Name: _____

Show all work clearly and in order **in separated papers**, and circle your final answers. This is a take-home quiz. Please hand in your solution in the discussion on October 21(Tuesday). This quiz is worth 25 points. 20 of them will be counted to your final score.

1. (6 points) Let a, b be constants. Verify the following formula.

$$\int e^{ax} \sin(bx) dx = \frac{e^{ax}}{a^2 + b^2} (a \cdot \sin(bx) - b \cdot \cos(bx)) + C$$

Hint: Use the strategy of integration by parts to $\int e^{ax} \sin(bx) dx$ twice. Compare the result and the original function.

2. (6 points) Let m and n be positive integers (both m and n are constants). Prove that

$$\int_{-\pi}^{\pi} \sin(mx) \sin(nx) dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$$

Hint: We have the following formula

$$\sin(a) \cdot \sin(b) = \frac{1}{2} [-\cos(a + b) + \cos(a - b)].$$

Take $a = mx, b = nx$. Apply the formula to the intergral and get the answer.

3. (7 points) Let a, b, c be positive constants. Solve the volume of the solid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1.$$

Hint: Recall disc mothod in Chapter 6.

Step 1: Determine x -axis.

Step 2: Find $A(x)$. Fixed x , we have

$$\frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 - \frac{x^2}{a^2}.$$

It is an ellipse on y - z coordinates. You should find its area. To do this, you need to write z in terms of y . Notice, a, b, c, x are all constants now. When solving the integral, Section 8.3 will help.

Step 3: $V = \int_{-a}^a A(x) dx$.

4. (6 points) Solve the integral

$$\int \frac{x^3 + 3x^2 + x}{x^3 + x^2 - 2} dx$$

Hint: We have the following equality

$$x^3 + x^2 - 2 = (x - 1)(x^2 + 2x + 2)$$