MATH141(0332) Calculus II

Quiz 10, Tuesday, November 18, 2008 Name: ______________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Calculator is not allowed in this quiz. You have 15 minutes to take this 10 point quiz.

1. (*3 points*) Find the radius of convergence R of the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots (2n+2)}{3 \cdot 8 \cdot 13 \cdots (5n+3)} x^{n-1}$$

2. (*3 points*) Given that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$. Find the Taylor series of $\frac{1}{(1-x)^2}$. $(-1 < x < 1)$
Hint: Notice that $\frac{d}{dx}\left(\frac{1}{1-x}\right) = -\frac{1}{(1-x)^2}$. You may use the Differentiation Theorem for Power Series.

3. (*4 points*) Given the Taylor series of $cos(x)$ to be

$$cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \cdots$$

We approximate the value of $cos(1)$ by $1 - \frac{1}{2} + \frac{1}{24} = \frac{13}{24}$. Use Lagrange Remainder Formula to give an explicit numerical upper bound of the error of this approximation.
Hint: $1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$ is the 5^{th} Taylor polynomial of $cos(x)$ around point 0. So, you should find an upper bound of $r_5(1)$.