

MATH101 Single Variable Calculus I

Summer 2016

Mid-term Exam

Name: _____

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Use no books, calculators, cellphones, communication with others, scratchpaper, etc. You have 120 minutes to take this 105 point exam. If you get more than 100 points, your grade will be 100.

Honor Pledge: The Rice University Honor Pledge reads:

"On my honor, I have neither given nor received any unauthorized aid on this exam."

Please write the exact wording of the Pledge, following by your signature, in the space below:

Pledge: _____

Your Signature: _____

Gradebox: For grading use. Please leave it blank.

Question:	1	2	3	4	5	6	7	Total
Points:	12	15	20	18	12	8	20	105
Score:								

Good Luck

1. (12 points) Mark each of the following statements T (True) or F (False).
You will get 2 points for each correct answer, -1 points for each wrong answer, and 0 point for leaving it blank.

(a) _____ If f and g are increasing in \mathbb{R} , then their product fg is also increasing in \mathbb{R} .

(b) _____ If f is an odd function in \mathbb{R} , g is an even function in \mathbb{R} , $g(x) \neq 0$ for all $x \in \mathbb{R}$.
Then, $\frac{f}{g}$ is an odd function in \mathbb{R} .

(c) _____ If $x > 0$ and $a > 1$, then $\frac{\ln x}{\ln a} = \ln \frac{x}{a}$.

(d) _____ If P is a polynomial, then $\lim_{x \rightarrow a} P(x) = P(a)$.

(e) _____ If f is differentiable, then $\frac{d}{dx} f(\sqrt{x}) = \frac{f'(x)}{2\sqrt{x}}$.

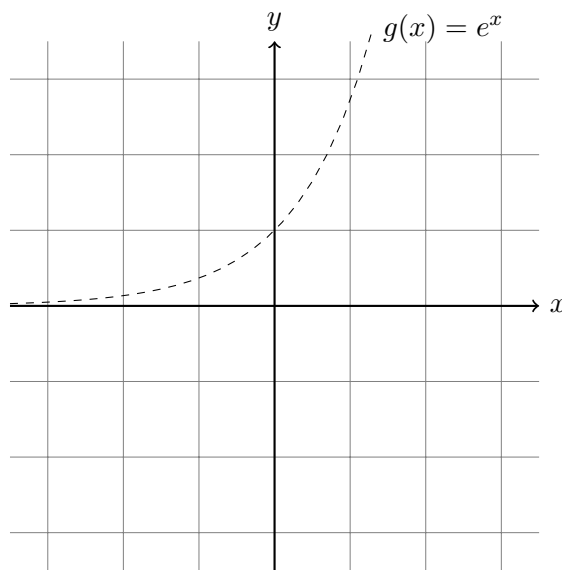
(f) _____ If f is continuous at $x = a$, then f is differentiable at $x = a$.

2. (15 points) Let $f(x) = 2e^{x-1} - 2$.

(a) What is the domain and range of f ?

(b) Starting from $g(x) = e^x$, under what transformations can we derive f ?

(c) Sketch the graph of f below. (The graph of g is given as a reference).



(d) Does f have an inverse? If so, write the algebraic representation of f^{-1} . What is the domain and range of f^{-1} ?

(e) Sketch f^{-1} in the same graph in (c). Mark the curve with f^{-1} to distinguish from the graph of f .

(f) Is f differentiable? Find its derivative f' .

3. (20 points) Denote $f \circ g$ as the composition of f and g .

(a) If $f(x) = \sqrt{x-3}$, $g(x) = x^2$, $h(x) = x^3 + 2$, find $f \circ g \circ h$.

(b) If $g(x) = 2x + 1$ and $h(x) = 4x^2 + 4x + 7$, find a function f such that $f \circ g = h$.

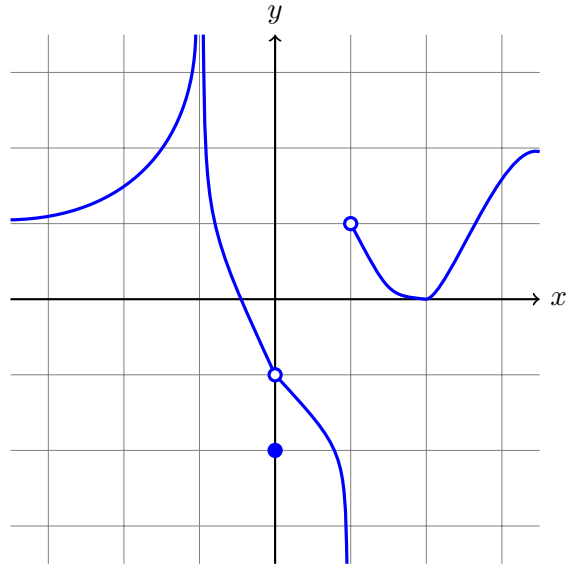
(c) If $f(x) = 3x + 5$ and $h(x) = 3x^2 + 3x + 2$, find a function g such that $f \circ g = h$.

(d) Let $A(x) = \sin^2(\cos x)$. Express f as composition of polynomials and trigonometric functions $f \circ g \circ h$. Write down f , g and h .

Find $A'(x)$.

(e) Find the limit $\lim_{x \rightarrow 0} \tan^{-1} \left(\frac{\sin x}{\tan x} \right)$.

4. (18 points) The graph of the function f is given as follows.



(a) What is the domain of the function?

(b) What is the range of the function?

(c) List all the vertical asymptotes.

(d) List all the horizontal asymptotes.

(e) Find the following limits (put DNE for “does not exist”).

$$\lim_{x \rightarrow -1^-} f(x) =$$

$$\lim_{x \rightarrow 0} f(x) =$$

$$\lim_{x \rightarrow 1} f(x) =$$

$$\lim_{x \rightarrow 2} f(x) =$$

$$\lim_{x \rightarrow -1^-} f(x) =$$

$$\lim_{x \rightarrow 0^-} f(x) =$$

$$\lim_{x \rightarrow 1^-} f(x) =$$

$$\lim_{x \rightarrow 1^+} f(x) =$$

(f) List all the points where f is not continuous?

5. (12 points) Evaluate the limits of the functions.

(a) $\lim_{x \rightarrow \infty} \frac{x^{20} + 2\sqrt{x} - 5}{2^x}$.

(b) $\lim_{x \rightarrow \infty} \frac{\sqrt{1 + 4x^6}}{2 - x^3}$.

(c) $\lim_{x \rightarrow 1^+} (\ln(x^2 + x - 2) - \ln(x - 1))$.

6. (8 points) Researchers measured the average blood alcohol concentration $C(t)$ of eight men starting one hour after consumption of 30 mL of ethanol (corresponding to two alcoholic drinks).

t (hours)	1.0	1.5	2.0	2.5	3.0
$C(t)$ (mg/mL)	0.33	0.24	0.18	0.12	0.07

(a) Find the average rate of change of C with respect to t over each time interval:

(i) [1.0, 2.0]

(ii) [1.5, 2.0]

(iii) [2.0, 2.5]

(iv) [2.0, 3.0]

(b) Estimate the instantaneous rate of change at $t = 2$ and interpret your result. What are the units?

7. (20 points) Differentiate the functions.

(a) $f(x) = x^{2.4} + e^{2.4}$.

(b) $f(x) = e^{-x}(4x^2 - 3\sqrt{x})$.

(c) $f(x) = \frac{\sin x}{1 + x \cos x}$.

(d) $f(x) = e^x \cos x$. Find both f' and f'' .

(e) Find the tangent line to $f(x) = \cos x - 5 \sin x$ at point $(\pi, -1)$.