STAT100 Elementary Statistics and Probability

Exam 3, Sample Test, Summer 2014 **Solution**

Instructions: Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. A correct answer with no justification may not receive full credit. The exam is worth a total of 104 points. The point value of each problem is indicated. The maximum score is 100.

Calculator is allowed in this exam for basic calculation only. It is also allowed to bring a piece of paper of formulas prepared by yourself.

1. A zoologist collected 10 wild lizards in the southwestern United States. The total length (mm) of each was measured.

   179 157 169 146 143 131 159 142 141 133.

   By calculation, we have sample mean $\bar{x} = 150$ and sample standard deviation $s = 15.61$. (You can use these values directly without verifying from the data.)

   (a) (8 points) Obtain a 95% confidence interval for the mean length.

   Solution: We use student t distribution with $d.f. = 9$. Look up the table and get $t_{.025} = 2.262$. **(4 points)** Therefore, the 95% confidence interval is

   $$\left[150 - 2.262\frac{15.61}{\sqrt{10}}, 150 + 2.262\frac{15.61}{\sqrt{10}}\right] = [138.83, 161.17]\,. \quad \textbf{(8 points)}$$

   (b) (4 points) Do these data provide strong evidence that the mean length is different from 145? Take $\alpha = .05$. (Answer by using your result in part (a).)

   Solution: The hypotheses read $H_0: \ \mu = 145$ vs $H_1: \mu \neq 145$. As 145 is inside the 95% confidence interval, the null hypothesis is retained. Therefore, the data do NOT provide strong evidence to support the argument. **(4 points)**

   (c) (10 points) Consider hypotheses $H_0 : \sigma = 18$ vesus $H_1 : \sigma < 18$. Should H_0 be rejected at $\alpha = .05$?

   Solution: We perform a χ^2 test. The test statistic is

   $$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{9 \cdot (15.61)^2}{18^2} = 6.77. \quad \textbf{(5 points)}$$

   We reject H_0 if $\chi^2 \leq \chi^2_{1-\alpha}$. Compute the cutoff $\chi^2_{.95} = 3.33$ with $d.f. = 9$. **(8 points)** As 6.77>3.33, we retain H_0. **(10 points)**

2. The data on the weight (lb) of male and female wolves, are given as

Female	57	84	90	71	71	77	68	74		
Male	71	93	101	84	88	117	86	93	86	81

(a) (10 points) Test the null hypothesis that the mean weight of **females** is 83 pounds versus a two-sided alternative. Take $\alpha = .05$.

> **Solution:** First, compute the sample mean and standard deviation for female wolves: $\bar{x} = 74$ and $s_1 = 10.06$. **(3 points)**
>
> Construct hypotheses: $H_0: \ \mu_1 = 80$ vs $H_1 : \mu_1 \neq 80$. We use t-test, rejecting H_0 if $|T| > t_{\alpha/2}$ with $d.f. = 7$. **(5 points)**
>
> Compute the test statistic $T = \dfrac{\bar{X} - \mu_1}{S_1/\sqrt{n}} = \dfrac{74 - 83}{10.06/\sqrt{8}} = -2.531$. **(7 points)**
>
> Look up the table to get the cutoff $t_{.025} = 2.365$ with $d.f. = 7$. **(9 points)**
>
> As $|-2.531| > 2.365$, we reject H_0. **(10 points)**

(b) (12 points) Obtain a 95% confidence interval for the difference of population mean weights between male and female wolves.

> **Solution:** Compute the sample mean and standard deviation for male wolves: $\bar{y} = 90$ and $s_2 = 12.39$. **(3 points)**
>
> The estimator of $\mu_1 - \mu_2$ is $\bar{x} - \bar{y} = -16$. **(4 points)**
>
> As $s1/s2 = .812$ is between $1/2$ and 2, we pool the data for standard deviation $s_{pooled} = \sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = 11.43.$
>
> The estimated standard error is $s_{pooled}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}} = 5.42$. **(7 points)**
>
> The cutoff is $t_{.025}$ is from t- distribution with $d.f. = n_1 + n_2 - 2 = 16$. Therefore, $t_{.025} = 2.120$. **(10 points)**
>
> The 95% confidence interval of $\mu_1 - \mu_2$ is
>
> $$[-16 - 2.120 \cdot 5.42, -16 + 2.120 \cdot 5.42] = [-27.49, -4.51]\,. \quad \textbf{(12 points)}$$

3. Given the following 2 data sets from 2 populations.

$$n_1 = 500, \quad \bar{x} = 24.4, \quad s_1^2 = 15.$$

$$n_2 = 300, \quad \bar{y} = 23.1, \quad s_2^2 = 18.$$

(a) (10 points) Find the 99% confidence interval of the difference in population means.

Solution: The estimator of $\mu_1 - \mu_2$ is $\bar{x} - \bar{y} = 1.3$. **(2 points)**

The estimated standard error is $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = .3$. **(5 points)**

As n_1, n_2 are big, we use Z-test. The cutoff is $z_{.005} = 2.575$. **(8 points)**

Finally, the 99% confidence interval is

$$[1.3 - 2.575 \cdot .3, 1.3 + 2.575 \cdot .3] = [.528, 2.073]. \quad \textbf{(10 points)}$$

(b) (8 points) Consider hypotheses $H_0 : \mu_1 - \mu_2 = 1$ vesus $H_1 : \mu_1 - \mu_2 > 1$. Find the P-value, and conclude that if the data support the argument H_1 or not.

Solution: First, compute the test statistic

$$z = \frac{\bar{x} - \bar{y} - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1.3 - 1}{.3} = 1. \quad \textbf{(3 points)}$$

Next, find the $P-$value

$$P - \text{value} = P(Z > z) = 1 - \Phi(1) = .1587. \quad \textbf{(6 points)}$$

As the $P-$value is big, the data does NOT support the argument H_1. **(8 points)**

4. A major clinical trial of a new vaccine for type-B hepatitis was conducted with a high-risk group of 1000 male volunteers. From this group, 600 men were given the vaccine and the other 400 a placebo. A follow-up of all these individuals yielded the data:

	Got hepatitis	Did not get hepatitis	Total
Vaccine	22	578	600
Placebo	58	442	400
Total	80	920	1000

To see if the vaccine is performing different from placebo, we form the hypotheses $H_0 : p_1 = p_2$ vesus $H_1 : p_1 \neq p_2$, where p_1, p_2 are proportions of people who get hepatitis by taking the vaccine or a placebo, respectively. Take $\alpha = .05$.

(a) (10 points) Use Z-test to determine if H_0 is rejected or not.

Solution: From the data, $\hat{p}_1 = \dfrac{22}{600} = .0367$, $\hat{p}_2 = \dfrac{58}{400} = .145$, $\hat{p} = \dfrac{80}{1000} = .08$.
The estimator of $p_1 - p_2$ is $\hat{p}_1 - \hat{p}_2 = -.1083$. **(2 points)**
The estimated standard error is $\sqrt{\hat{p}(1-\hat{p})}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}} = .0175$. **(5 points)**
The test statistic $z = \frac{-.1083}{.0175} = -6.19$. **(7 points)**
Two sided cutoff $z_{.025} = 1.96$. **(8 points)**
As $|-6.19| > 1.96$, we reject H_0. **(10 points)**

(b) (10 points) Use χ^2-test to determine if H_0 is rejected or not.

Solution: Compute the expected frequency. **(4 points)**

	Got hepatitis	Did not get hepatitis	Total
Vaccine	22	578	600
	(48)	(552)	
Placebo	58	442	400
	(32)	(368)	
Total	80	920	1000

Perform a χ^2 test **(8 points)**

	Got hepatitis	Did not get hepatitis	
Vaccine	14.083	1.225	
Placebo	21.125	1.837	
			$\chi^2 = 38.27$ $d.f. = 1$

The cutoff for χ^2 test with degree of freedom 1 is $\chi^2_{.05} = 3.84$.
As $38.27 > 3.84$, we reject H_0. **(10 points)**

5. A survey was conducted by sampling 400 persons who were questioned regarding union membership and attitude toward decreased national spending on social welfare programs. The cross-tabulated frequency counts are presented.

	Support	Indifferent	Opposed	Total
Union	110	30	20	160
Nonunion	90	70	80	240
Total	200	100	100	400

(a) (10 points) Consider *only* people inside the union. Test the null hypothesis that the probability of support, indifferent and opposed is in the ratio 4:1:1. Use $\alpha = .05$.

Solution: Use a χ^2 test. **(7 points)**

	Support	Indifferent	Opposed	Total
Observed O	110	30	20	160
Prob. under H_0	4/6	1/6	1/6	1
Expeced freq. E	106.667	26.667	26.667	160
$(O-E)^2/E$	.104	.417	1.667	$\chi^2 = 2.188$ $d.f. = 2$

The cutoff for χ^2 test with degree of freedom 2 is $\chi^2_{.05} = 5.99$. **(9 points)**
As $2.188 < 5.99$, we retain the null hypothesis. **(10 points)**

(b) (12 points) Do these data imply that the attitude and membership are independent or associated? Use $\alpha = .05$.

Solution: Compute the expected frequency **(4 points)**

	Support	Indifferent	Opposed	Total
Union	110 (80)	30 (40)	20 (40)	160
Nonunion	90 (120)	70 (60)	80 (60)	240
Total	200	100	100	400

Perform a χ^2 test **(8 points)**

	Support	Indifferent	Opposed	Total
Union	11.25	2.5	10	
Nonunion	7.5	1.667	6.667	
				$\chi^2 = 39.584$ $d.f. = 2$

The cutoff for χ^2 test with degree of freedom 2 is $\chi^2_{.05} = 5.99$. **(10 points)**
As $39.584 > 5.99$, we reject H_0. Therefore, the attitude and membership are NOT independent. **(12 points)**