

STAT100 Elementary Statistics and Probability Summer II 2014

Quiz 6, Friday, August 1, 2014

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Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. You are allowed to calculator for basic calculation in this quiz. You have 10 minutes to take this 10 point quiz.

Z is a random variable with standard normal distribution. X also has a normal distribution with mean 2 and variance 9.

1. (3 points) Find $P(|Z| \leq 1)$.

$$\begin{aligned}P(|Z| \leq 1) &= P(-1 \leq Z \leq 1) = \Phi(1) - \Phi(-1) \\ &= \Phi(1) - (1 - \Phi(1)) \\ &= 2\Phi(1) - 1 = 2 \times .8413 - 1 \\ &= .6826\end{aligned}$$

2. (3 points) Find the 30th percentile of the standard normal distribution.

$$\begin{aligned}\text{30th percentile: } & \Phi^{-1}(.3) \\ \text{By symmetry: } & = -\Phi^{-1}(1-.3) = -\Phi^{-1}(.7) \\ \text{As } \Phi(.52) &= .6985 \approx .7 \Rightarrow \Phi^{-1}(.7) = .52 \\ \Rightarrow \Phi^{-1}(.3) &= -.52\end{aligned}$$

3. (4 points) Find $P(X \geq 0)$.

$$\begin{aligned}X \sim \mathcal{N}(2, 9) &\Rightarrow Z = \frac{X-2}{3} \sim \mathcal{N}(0, 1) \\ P(X \geq 0) &= P\left(\frac{X-2}{3} \geq \frac{0-2}{3}\right) = P\left(Z \geq -\frac{2}{3}\right) \\ &= 1 - \Phi\left(-\frac{2}{3}\right) = 1 - (1 - \Phi\left(\frac{2}{3}\right)) = \Phi\left(\frac{2}{3}\right) = .7486 \\ & \qquad \qquad \qquad \uparrow \\ & \qquad \qquad \qquad .67\end{aligned}$$