STAT100 Elementary Statistics and Probability Summer II 2014

Quiz 10, Friday, August 15, 2014 Name: Jeff

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. You are allowed to calculator for basic calculation in this quiz. You have 15 minutes to take this 10 point quiz.

1. *(5 points)* A set of data has the following summary statistics

$$n = 30, \quad \bar{x} = 25.35 \quad s = 2.15.$$

Do the data substantiate the conjecture that $\sigma < 2.2$? Test at $\alpha = .05$.

First, the test statistic is $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$ with d.f. $= n-1$

We have a one-side test $H_0: \sigma = 2.2$ $H_1: \sigma < 2.2$.

From the data, we get $\chi^2 = \frac{29 \cdot 2.15^2}{2.2^2} = 27.697$

The regection region is $\chi^2 \le \chi^2_{1-\alpha} = 17.71$ ← Look up from the table with d.f = 29

As $27.697 > 17.71$, we retain H_0.

Therefore, the data does not substantiate $\sigma < 2.2$.

2. *(5 points)* Given the following 2 data sets from 2 populations.

Rmk. We can also use the relation between confidence interval and two-side test hypothesis.

$n_1 = 300, \quad \bar{x} = 17.2 \quad s_1^2 = 18.$ large sample size!

$n_2 = 500, \quad \bar{y} = 17.5 \quad s_2^2 = 15.$

Find the 95% confidence interval of the difference in population means. Use the result to determine whether the conjecture that the 2 populations have different means is supported from the data. (Take $\alpha = .05$). *Hint: Use the relation between confidence interval and two-sided testing hypotheses to determine that rejecting or retaining the null hypothesis.*

① Estimator $\bar{X} - \bar{Y} = -.3$ Standard error. $\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = .3$

$\alpha = .05 \Rightarrow z_{\alpha/2} = z_{.025} = 1.96$

$\Rightarrow$ 95% Confidence interval of $\mu_1 - \mu_2$ is $[-.3 - 1.96 \times .3, -.3 + 1.96 \times .3] = [-.888, .288]$

② Hypothesis test: $H_0: \mu_1 - \mu_2 = 0$, $H_1: \mu_1 - \mu_2 \neq 0$

Test statistic: $Z = \frac{\bar{X} - \bar{Y} - 0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{-.3 - 0}{.3} = -1$

Rejection region: $|Z| \ge z_{\alpha/2}$ where $z_{\alpha/2} = z_{.025} = 1.96$ ← by data.

As $|-1| < 1.96$, we retain H_0. $\Rightarrow$ The conjecture is NOT supported