

# STAT100 Elementary Statistics and Probability

Exam 1, Monday, July 28, 2014

Name: \_\_\_\_\_

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Calculators are allowed in this exam. You have 80 minutes to take this 106 point exam. If you get more than 100 points, your grade will be 100.

1. (16 points) The city of Madison regularly checks the quality of water at swimming beaches located on area lakes. Fifteen times of concentration fecal coliforms, in number of colony forming unites (CFU) per 100 ml of water, was measured during the summer at one beach.

180 1600 90 140 50 260 400 90 380 110 10 60 20 340 80

- (a) (6 points) Calculate the sample mean and median.
- (b) (6 points) One day, the water quality was bad - the reading was 1600 CFU - and the beach was closed. Drop this value and calculate the sample mean and median for the days where water quality was suitable for swimming.
- (c) (4 points) Which one is more robust: sample mean or median? Explain why.
2. (24 points) Consider the following data set.

$x$	2	3	5	7	8
$y$	9	7	5	3	3

- (a) (10 points) Compute the correlation coefficient between  $x$  and  $y$ . Do they have a strong linear relation?
- (b) (10 points) Find the equations of the least squares fitted line.
- (c) (4 points) Using the fitted line, predict the value of  $y$  when  $x = 4$ .
3. (20 points) A table containing probabilities of two events  $A$  and  $B$  is given in the right.

	$A$	$\bar{A}$	
$B$	.12		.4
$\bar{B}$			
		.7	

- (a) (4 points) Fill in the missing entries of the table.
- (b) (4 points) Find the probability  $P(A\bar{B})$ .
- (c) (4 points) Find the probability  $P(A|\bar{B})$ .
- (d) (4 points) Are  $A$  and  $\bar{B}$  disjoint? Explain Why.
- (e) (4 points) Are  $A$  and  $\bar{B}$  independent? Explain Why.

4. (24 points) In a shipment of 12 room air conditioners, there are 3 with defective thermostats. Two air conditioners will be selected at random and inspected one after another. Denote events

$$A = \{\text{The first one is defective}\},$$

$$B = \{\text{The second one is defective}\}.$$

Express the following events using in set notations and find their probabilities.

- (a) (4 points) The first is defective.
- (b) (4 points) The second is good given that the first is defective.  
*Hint: if the first one is defective, then there are 11 left, with 2 defective ones.*
- (c) (4 points) The first is defective and the second is good.
- (d) (6 points) The first is defective given the second is good.
- (e) (6 points) Exactly one is defective.
5. (12 points)  $X$  is a discrete random variable. Its distribution is given as below.

$x_i$	0	1	2	3
$f(x_i)$	1/64	9/64	27/64	27/64

- (a) (4 points) Find  $\mathbb{E}X$ .
- (b) (4 points) Find  $\text{Var}(X)$  and  $\text{sd}(X)$ .
- (c) (4 points) Suppose  $Y = 4X + 3$ . Find  $\mathbb{E}Y$  and  $\text{Var}(Y)$ .
6. (10 points) Suppose 15% of the trees in a forest have severe leaf damage from air pollution. If 5 trees are selected at random, find the probability that no more than two have severe leaf damage.