STAT100 Elementary Statistics and Probability

Exam 2, Monday, August 6, 2012 Name:________________________

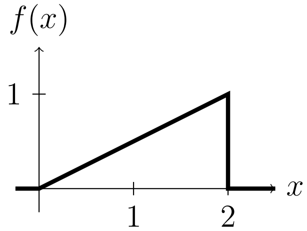
Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Calculators are allowed in this exam for basic calculation only. You have 80 minutes to take this 104 point exam. If you get more than 100 points, your grade will be 100.

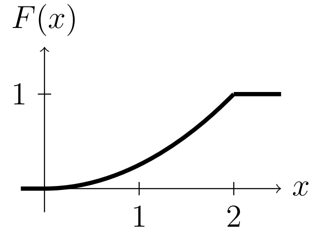
1. (24 points)

   (a) (12 points) Last year, 75% of UMD students read the Diamondback. In a group of 1200 randomly selected students, what is the approximate probability that fewer than or equal to 880 students read the Diamondback?

   (b) (12 points) To determine if this percentage has changed, a random sample of 300 students is studied and 246 read the Diamondback. Does this finding indicate that the current percentage of students reading the Diamondback has changed from what is was last year? (Form the testing hypotheses, compute the $P-$value and determine if the percentage has changed or not based on the data.)

2. (20 points) A continuous random variable X has the following probability density function f and cumulative distribution function F.





$$f(x) = \begin{cases} x/2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases} \qquad F(x) = \begin{cases} 0 & x \le 0 \\ x^2/4 & 1 \le x \le 2 \\ 1 & x \ge 2 \end{cases}$$

   (a) (5 points) Which of the two intervals $[0 < X < .5]$ or $[1.5 < X < 2]$ is assigned a higher probability?

   (b) (5 points) Compute $P(.5 \le X \le 1.5)$.

   (c) (5 points) Determine the median of X.

   (d) (5 points) Is X left-skewed, right-skewed or symmetric?

3. (20 points)

   (a) (5 points) Φ is the cumulative distribution function of standard normal distribution. Find $\Phi(1.222)$ and $\Phi(1.342)$. Use the normal table and linear interpolation to get your answer. (Round up to 4 digits)

(b) (5 points) Suppose Z is a random variable with standard normal distribution. Find $P(|Z| \leq 1.222)$.

(c) (5 points) Suppose X has a normal distribution with mean 4 and standard deviation 2. Find $P(6.444 \leq X \leq 6.684)$.

(d) (5 points) [*] Suppose Y has a normal distribution with mean -6.41 and standard deviation 5. Find $P(|Y| > .3)$.

4. (20 points) The number of complaints per day, X, received ny a cable TV distributor has the probability distribution

x	0	1	2	3
$f(x)$	.4	.3	.1	.2

(a) (6 points) Find the expectation and standard deviation of the number of complaints per day.

(b) (6 points) Find the expectation and standard deviation of the average number of complaints per day in 90 days. *(i.e., find* $\mathbb{E}\bar{X}$ *and* $Sd(\bar{X})$*, where* $\bar{X} = (X_1 + \cdots + X_{90})/90$*.)*

(c) (8 points) What is the approximate probability that the distributor will receive more that 125 complaints in 90 days.

5. (20 points) A credit company randomly selected 60 contested items and recorded the dollar amount being contested. These contested items had sample mean $\bar{x} = 75$ dollars and sample variance $s^2 = 9$.

(a) (4 points) Give a point estimate of the population mean contested amount μ.

(b) (6 points) Calculate a 90% confidence interval for the population mean contested amount.

(c) (10 points) We formulate the following hypotheses

$$H_0 : \ \mu = 74 \quad \text{versus} \quad H_1 : \mu > 74.$$

Take $\alpha = .008$ (meaning that the probability of making type I error is less than .8%). What does the data suggest, rejecting H_0 or retaining H_0?