

STAT100 Elementary Statistics and Probability

Exam 1, Monday, July 23, 2012

Solution

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Calculators are allowed in this exam. You have 80 minutes to take this 104 point exam. If you get more than 100 points, your grade will be 100.

1. (16 points) The following measurements of the diameters (in feet) of Indian mounds in southern Wisconsin were gathered by examining reports in the *Wisconsin Archeologist*.

22 24 24 30 22 20 28 30 24 34 36 15 37

- (a) (8 points) Calculate the mean and median.
(b) (8 points) Calculate the sample range and the interquartile range.
2. (24 points) Consider the following data set.

x	1	2	7	4	6
y	5	4	2	3	2

- (a) (10 points) Compute the correlation coefficient between x and y . Do they have a strong linear relation?
(b) (10 points) Find the equations of the least squares fitted line.
(c) (4 points) Using the fitted line, predict the value of y when $x = 5$.
3. (20 points) For two events A and B , the following probabilities are given.

$$P(A) = .4 \quad P(B) = .25 \quad P(A|B) = .7$$

Use the appropriate laws of probability to calculate

- (a) (4 points) $P(\bar{A})$.
(b) (4 points) $P(AB)$.
(c) (4 points) $P(A \cup B)$.
(d) (4 points) $P(A\bar{B})$.
(e) (4 points) Are the two events independent? Why or why not?
4. (20 points) X is a discrete random variable. Its distribution is given as below.

x_i	0	1	2	3
$f(x_i)$	27/64	27/64	9/64	1/64

- (a) (4 points) Find $\mathbb{E}X$.

- (b) (4 points) Find $\text{Var}(X)$ and $\text{sd}(X)$.
- (c) (6 points) Suppose $Y = 4X - 3$. Find $\mathbb{E}Y$ and $\text{Var}(Y)$.
- (d) (6 points) [*] The *skewness* of a random variable X is defined by

$$\text{Skew}(X) = \mathbb{E} \left[\left(\frac{X - \mu}{\sigma} \right)^3 \right], \text{ where } \mu = \mathbb{E}X \text{ and } \sigma = \text{sd}(X).$$

Find $\text{Skew}(X)$.

5. (24 points) An urn contains two green balls and three red balls.
- (a) (12 points) Suppose two balls will be drawn at random one after another and *without* replacement (i.e., the first ball drawn is *not* returned to the urn before the second one is drawn). Find the probabilities of the events

$$A = \{\text{Red ball appears in the first draw}\}$$

$$B = \{\text{Red ball appears in the second draw}\}$$

- (b) (12 points) Suppose four balls will be drawn at random *with* replacement (i.e. The ball drawn will be returned to the urn before the next one is drawn). Find the probability that at least two red balls are drawn.